Adaptive Bayesian Shrinkage Model for Spherical Wavelet Based De-noising of the Hippocampus

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http://picsl.upenn.edu/caph08/papers/paper11.pdf

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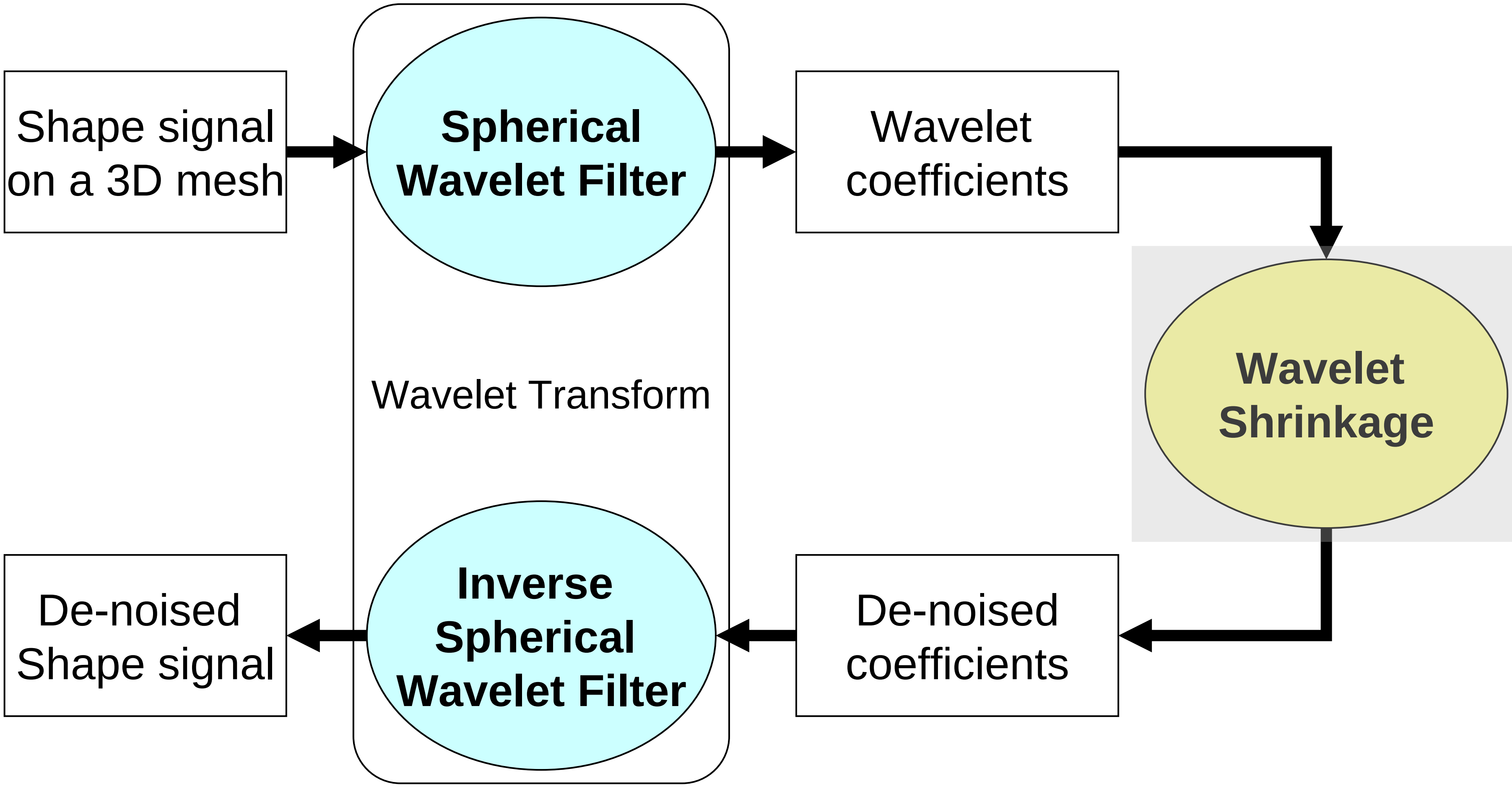
Overview

We present a statistical wavelet-based model for shape de-noising and compression

- Shape signal is encoded using spherical wavelets on a multiresolution mesh (*Nain et al. MICCAI, 2005*)
- We propose an adaptive Bayesian shrinkage model to remove wavelet coefficients that likely correspond to noise
- Our shrinkage scheme integrates local shape information as it takes into account local curvature and neighboring vertices

Processing Flow



Wavelet Shrinkage

- Wavelet shrinkage is a signal de-noising technique based on the idea of thresholding high frequency wavelet coefficients that correspond to irregularities in the shape signal

- Our shrinkage consists in a hard thresholding where:
  - Coefficients corresponding to irregularities/noise are “shrunk to 0”
  - We want to obtain a noiseless version of the shape without losing any relevant information
  - We want to be able to encode the shape signal with a high compression rate (ie with few coefficients)

Shape Model and Wavelet Transform

The observed shape signal is represented as a 3-N vector $\vec{y}$

$$\vec{y} = \vec{f} + \vec{\varepsilon}$$

Signal of interest   Gaussian noise

From the linearity of the wavelet transform, we obtain a vector of wavelet coefficients d

$$\vec{d} = \vec{\theta} + \vec{\epsilon}$$

Coefficients to be estimated   Gaussian noise coefficients

Hypothesis Testing and Shrinkage Rule

- Coefficient thresholding is based on **hypothesis testing:**

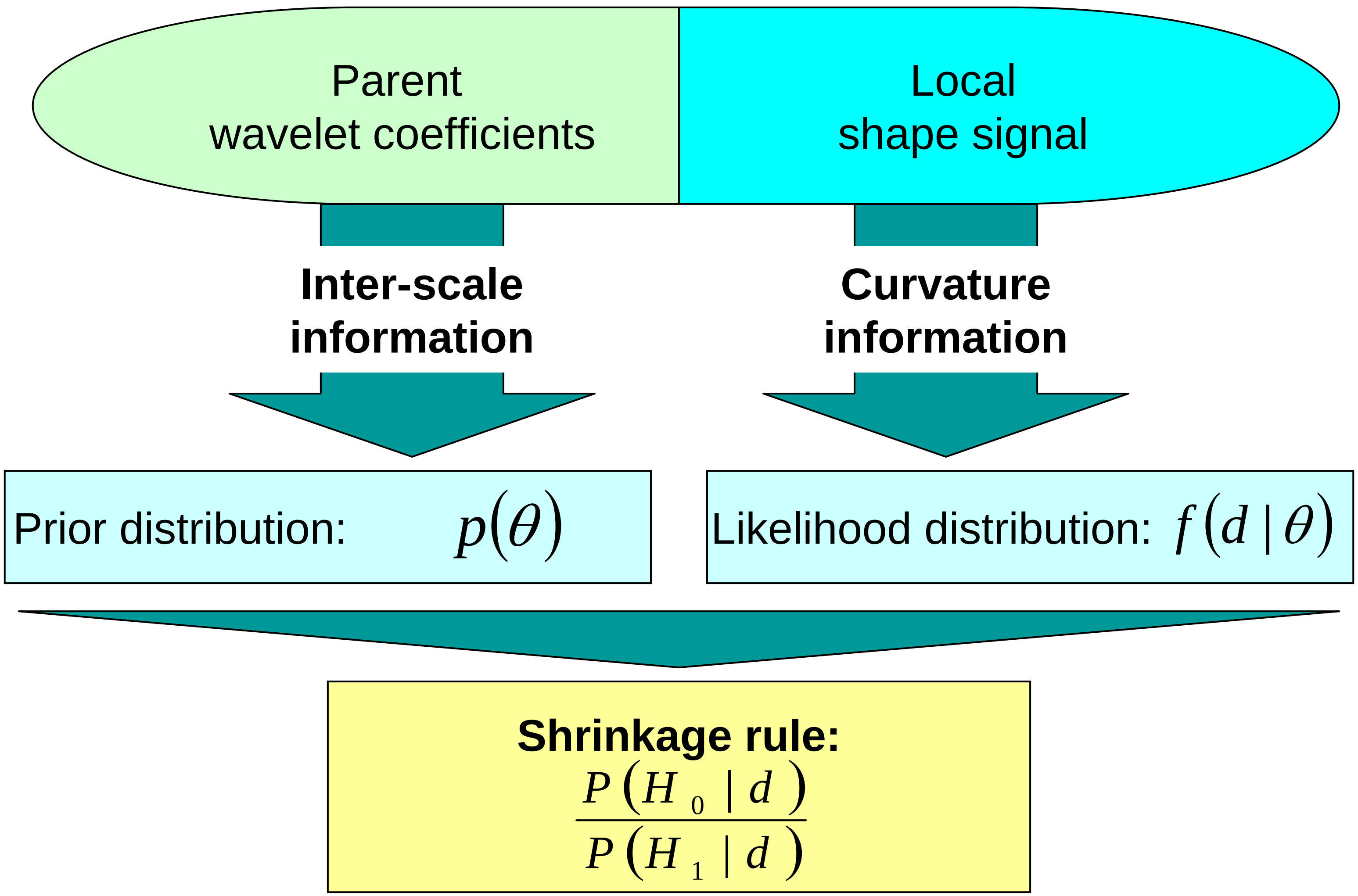
$$H_0 : \theta = 0$$

- If rejected, we set $\theta = d$ as the signal part coefficient, where d and θ refer to an arbitrary coefficient

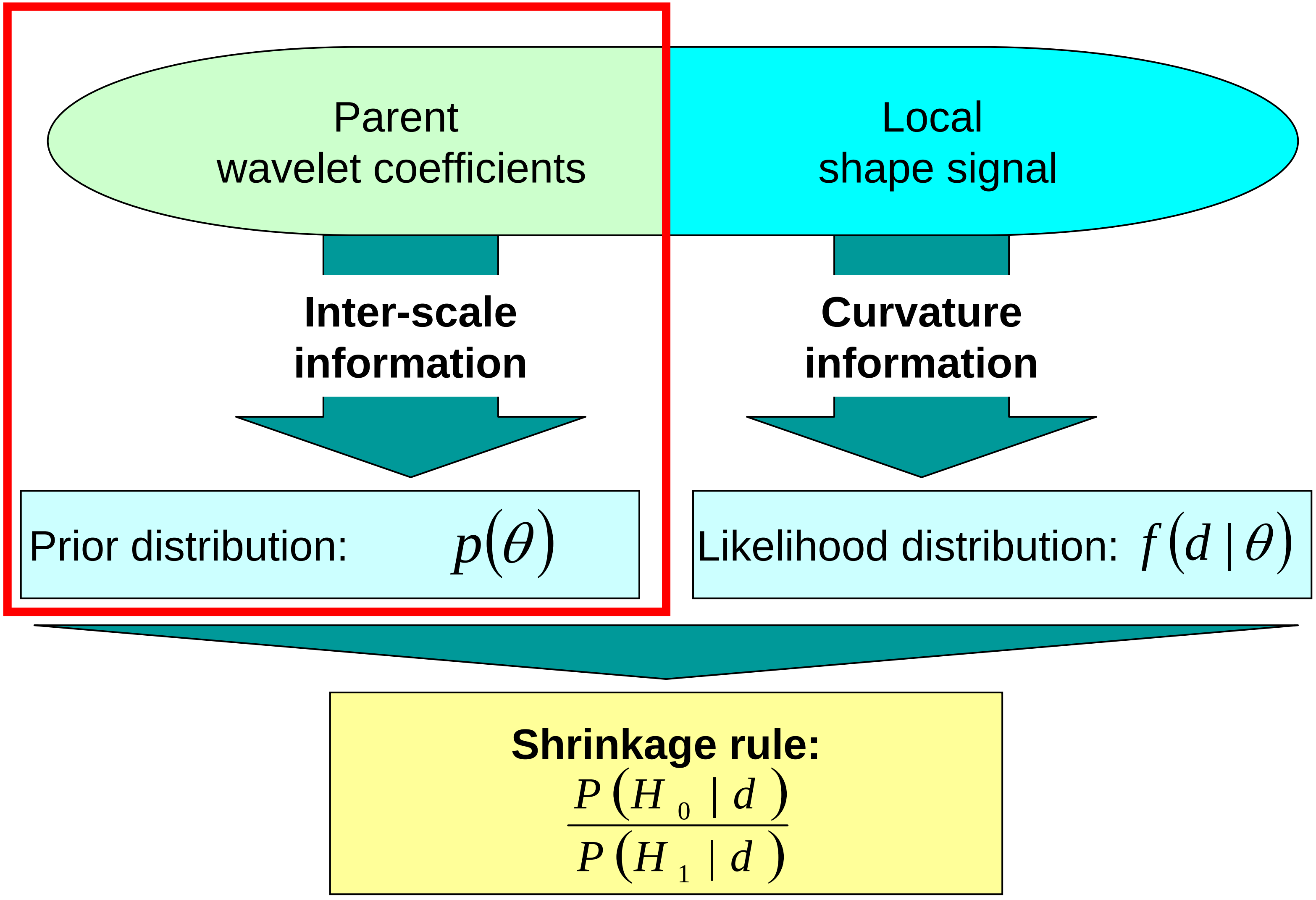
- For each wavelet coefficient subject to shrinkage, we estimate the posterior odds in favor of H_0:

$$\frac{P(H_0 \mid d)}{P(H_1 \mid d)} = \frac{P(\theta = 0 \mid d)}{P(\theta \neq 0 \mid d)}$$

A Data-driven Bayesian Framework



Prior Distribution (1/2)



Prior Distribution (2/2)

- We want to recover a smooth and accurate shape signal
- Relevant information brought by neighbors from coarser or/and same levels

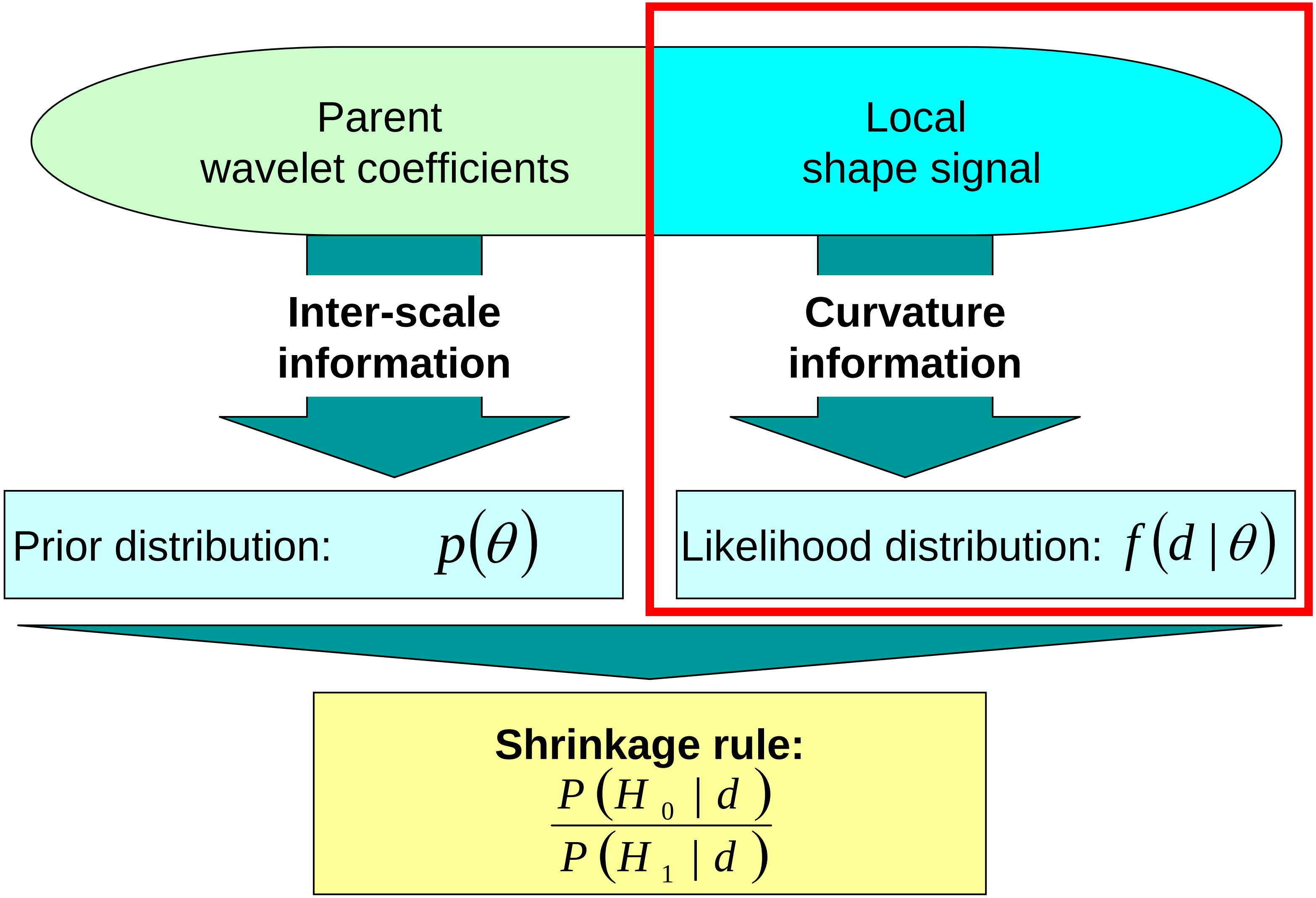
- Prior distribution is modeled as the mixture of a point mass at zero with a spread distribution. Mixture weights contain neighbor information from coarser levels

$$p(\theta) = \pi_0(neighbors).\delta_0 + \pi_1(neighbors).\xi(\theta)$$

$$\pi_0 + \pi_1 = 1$$

- π_0 is a decreasing function of the neighborhood magnitude

Likelihood Distribution (1/2)



Likelihood Distribution (2/2)

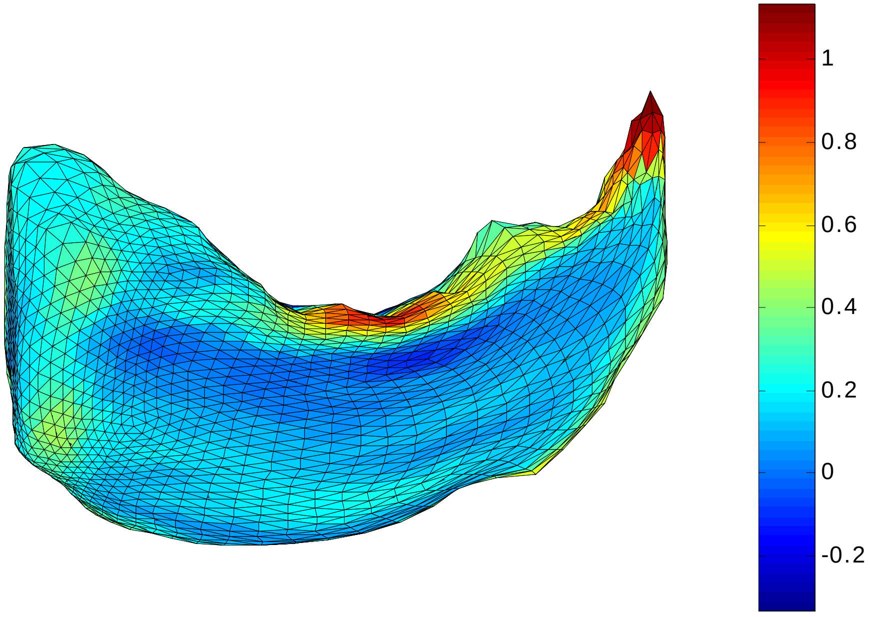
- Likelihood integrates curvature information κ
- Lower curvature increases shrinkage
- Noise is assumed to be Gaussian

$$f(d|\theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-\mid d \cdot \kappa - \theta \mid^2}{\sigma^2}\right)$$

- κ is between 0 and 1 and is computed at each vertex. It consists of the maximum absolute value of the two principal curvatures

Curvature Information

- Curvature is computed on the mesh (Taubin, 95)
- The shape is first smoothed through “linear” shrinkage



κ on the shape

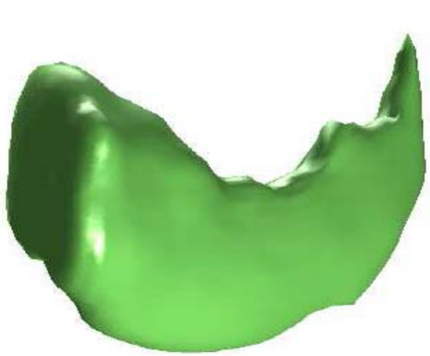
Implementation

- Coefficients are clustered by scale and analysis is run level by level, from coarser to finer
- Shrunk values of neighbors from coarser levels (1 to j) are used at level (j+1)
- Only fine levels are assumed to be affected by noise
- The 3 coordinates are analyzed separately

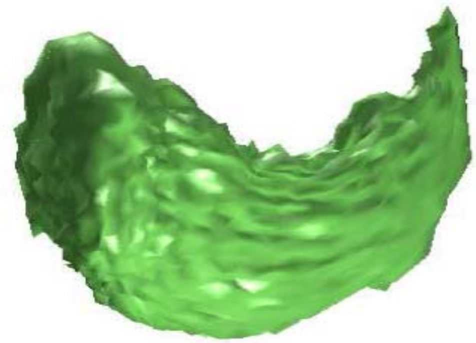
Experiments

- We test our method on a data set of 25 left Hippocampus shapes to which we have had Gaussian noise
- We compare our results to:
  - Universal and other types of classical thresholding (Donoho) with threshold $\lambda = \sqrt{k \cdot \log N} \cdot \sigma$
  - Previously published results using only inter-scale information

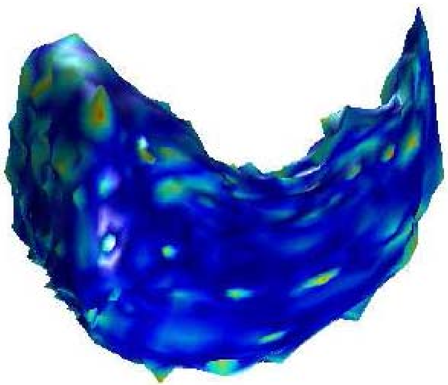
Hippocampus De-noising Results



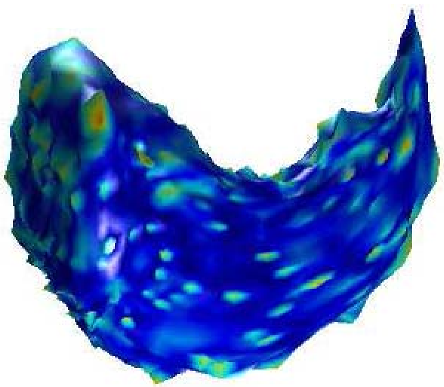
Original shape



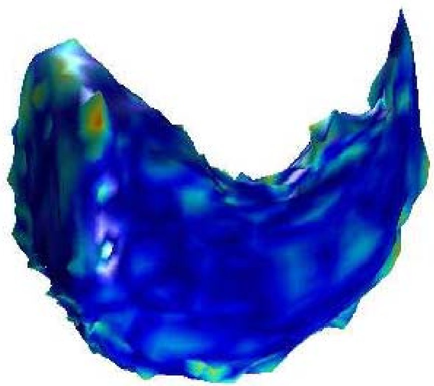
Noisy shape



De-noised shape with classical thresholding



De-noised shape with inter-scale thresholding



De-noised shape with proposed thresholding

Conclusion

- Multiscale analysis allows for a very powerful signal representation
- We are able to remove a large number of spherical wavelet coefficients without losing relevant information
- Bayesian methods enable to incorporate useful information into the model (neighbors, curvature)
- Parameter estimation remains a key issue
- Future work includes comparison to classical smoothing techniques, applications to noisy MR images and sharpening of the model (prior for noise variance…)